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Theobald Boehm and the Scale of the Modern Flute

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THE FLUTE as we know it today is essentially the instrument created and introduced by Theobald Boehm in 1847. Prior to the introduction of this instrument, Boehm had spent much time studying acoustics with Prof. Karl von Schafhäutl and making many experiments to determine optimal acoustic proportions, including the shape of the bore, the size and location of fingerholes, and the geometry of the mouth hole. The improvements he made, in combination with key mechanisms designed for easy fingering, produced an instrument which has called for very little change over the succeeding 135 years.

During the last decade, it has become more and more widely recognized that flutes in common use have a tendency to be flat for notes at the low end of the instrument and sharp for those at the upper end—in effect that the spacing of the tone holes is somewhat greater than proper intonation calls for. Many flute makers are now producing "new scale" instruments in which this spacing has been slightly reduced. It has been suggested that this effect was a result of a change in standard pitch from A–435 to A–440, occurring in the early part of the twentieth century, and that many flute makers changed the instrument merely by cutting off the head joint, leaving a body that was too long. In fact, however, it is easy to show that Boehm's own design methods, and the actual instruments he produced, suffer from this same fault, which has persisted to some extent in the design of the instrument over the entire period. It is the purpose of this article to trace the evolution of Boehm's flute design with respect to the location of the tone holes, and to show how this has changed with time.

The placement of the holes in the body of the flute in order to produce a proper scale is analogous to the placing of the frets on a guitar. The latter problem has a straightforward solution—the octave of the open-string note is produced when the string is shortened to one-half its length, and the frets for intervening notes are placed so that each successive fret shortens the string by a factor of one divided by the twelfth root of two. The problem for the flute is not so simple. We may liken it to the imaginary problem of designing a guitar whose string is not uniform in mass along its length: not only does it taper at one end, but it is "lumpy" in addition. Then our hypothetical guitar designer is faced with a bridge position that varies during the performance. He also finds that his fret does not stop the string vibration suddenly, but that some of the vibration persists beyond the finger to the next fret. And finally, he must play notes in the higher range, lumpy string notwithstanding, by using "harmonics," with the string vibrating in segments.

In spite of these additional variables on the flute (which have their origins, respectively, in the nonuniform temperature of the air column, the cavities formed by the closed keys, the changing coverage of the mouth hole by the performer's lips, and a finger hole that opens to the side but does not cut off the rest of the bore), the concept of a set of hole positions corresponding to the ends of equivalent string lengths is a reasonably valid starting point. One must, however, determine (usually empirically) equivalent points from which to measure the length, at both the mouth-hole end and the finger-hole end of the air column. It is the determination of this "basic length," and of the end corrections to be applied, that largely distinguishes one design technique from another.

Boehm published his findings extensively. He was anxious that others would understand not only the results of his work, but also the methods by which he approached the problem. His first major publication[1] described in considerable detail his work on conical flutes as well as the Boehm cylinder flute. In 1862 he sent to the World's Exposition in London a description of his "schema," a geometrical construction by means of which a maker could readily derive the positions of the holes for flutes designed to different pitches. In 1867 the schema was again submitted to a World's Exposition in Paris, this time with somewhat altered dimensions. An extensive description of this was published in 1868.[2] In 1871 Boehm published his well-known book *Die Flöte und das Flötenspiel,*[3] in which he reviewed selected portions of the 1847, 1862,

1. Theobald Boehm, *Über den Flötenbau und die neuesten Verbesserungen desselben* (Mainz, 1847; reprint ed., Buren: Frits Knuf, 1981).

2. Theobald Boehm, "Über die Bestimmung der Tonlöcherstellung auf Blasinstrumenten in beliebig verschiedenen Stimmungen," *Kunst- und Gewerbeblatt des Polytechnischen Vereins für das Königreich Bayern* (Munich, 1868), cols. 579–86.

3. Theobald Boehm, *Die Flöte und das Flötenspiel in akustischer, technischer und artistischer Beziehung* (Munich, 1871), trans. Dayton C. Miller, *The Flute and Flute-Playing in Acoustical, Technical, and Artistic Aspects* (2d ed. rev., 1922; reprint ed., New York: Dover Publications, 1964).

and 1868 works. The schema included in this work is the 1867 version.

Besides the published works, there exist today many instruments of Boehm's manufacture which are available for measurement. The Dayton C. Miller Flute Collection at the Library of Congress in Washington, D.C., contains thirty-nine specimens of Boehm or Boehm and Mendler manufacture. Also preserved there are measurements made by Miller of Boehm flutes not in his collection, so that altogether there is a seemingly rich source to explore for understanding the changes which took place in specification of the flute dimensions.

Unfortunately, the search is considerably hampered by the paucity of information on the intended pitch of the instruments, and by a surprising vagueness in much of Boehm's work as to just what pitch he was working to at a given time. A transcription of Boehm's record book of the manufacture and distribution of all his cylinder flutes, from no. 1 in 1847 to April, 1879, briefly describes each flute, but does not give the design pitch of any instrument. A search of Boehm's correspondence kindly made for me by his great-great-grandson, Ludwig Böhm,[4] finds nothing but a few offers to supply flutes of the "new Paris pitch." The pitches given by Miller for instruments in his collection are evidently his own estimates, and they leave a somewhat tenuous connection between what Boehm prescribed (which changed with time) and what he actually intended to do in the manufacture of a particular instrument.

Boehm's Principles for Determining Fingerhole Location

In his earliest experiments with the cylinder flute, Boehm determined an optimal internal diameter (19 mm) for the cylindrical portion of the tube, a taper for the head joint, and dimensions of the embouchure hole and stopper cavity. These were chosen to give the best tonal response and to maintain a good approximation of an octave on overblowing to attain the second register. The dimensions of these parameters are, from all subsequent experience, so close to providing the best compromise that we find very little departure from them even today. Boehm kept them essentially fixed for almost all of his C flutes. While some makers have varied the taper of the head joint, most of the head-joint tapers used today differ from Boehm's to a degree that is insignificant in determining the intonation properties of the instrument.

4. Personal letter from Ludwig Böhm, Munich, November 13, 1982.

Boehm's experiments with simple tubes showed that the length required for a tube open at both ends to resonate, say at low *c*′, was somewhat longer than that for the flute, a difference which Boehm properly ascribed to the taper, the geometry around the mouth hole, and the effect of blowing. Moreover, a simple tube cut in half did not resonate exactly one octave higher. Here, Boehm, following Schafhäutl, mentions the change in diameter-to-length ratio as a cause. Today we would more exactly ascribe the cause to an end correction; the effective end of the tube lies at a point about 0.3 of its diameter beyond the physical end. This concept of an end correction, either for the simple tube or the more complex mouth-hole geometry, was not well understood in Boehm's day. It was clear to Boehm, however, that he could not take the simple physical length of the flute and treat it like the guitar string. The problem was to find its "basic length," that is, the length it would have if one could define its ends as simply as those of a string, and to fix a point from which to measure all the tone-hole centers so that the low *c*′ hole was twice the distance from this point as the hole which produced the *c*″ an octave above. (This statement refers to a *c*′ hole in a flute going to low *b*, to avoid the complication, of which Boehm was quite aware, of the different effect of a tone hold from a cut-off end.)

Figure 1 shows these essential parameters. The use of the cork as representing the end of the actual flute has no acoustical justification, though Boehm attempted in his early publication to assign one. Since it is standardly placed 17 mm from the center of the mouth hole, it will do as well as any for a reference point.

Once having decided to place the tone-hole centers at their theoret-

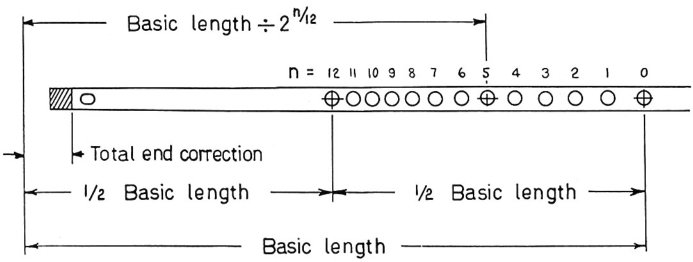

FIGURE 1. Schematic arrangement of the major tone holes of the flute, as in Boehm's string model. Distances are measured from a reference point lying beyond the cork by the amount of the total end correction.

ical "string length" distances, Boehm's design problem became one of making a determination of the basic length, and of the distance to a reference point beyond the cork, which we will call the "total end correction." We lump the end corrections for both ends of the flute into this quantity, as Boehm did.

The "stretched scale" of many flutes is largely a result of using too long a basic length, and the present article centers its attention on what Boehm prescribed for a basic length and how he arrived at the prescription.

It must not be assumed that merely by getting these two parameters right, the flute would be perfectly tuned. There are several variables which are not encompassed by these concepts.[5] The total end correction is in fact not constant as the frequency changes, though Boehm's invention of the tapered head goes a long way to making it approach a constant in spite of the necessary change in lip coverage of the mouth hold with frequency. There is a nonuniform distribution of temperature, and therefore sound velocity, along the tube. The cavities formed by the tone holes when they are closed alter the effective length of the tube, but not in the same way in the second octave as in the first. And especially in the third octave, the effects of the tube beyond the first open hole and the effects of vent holes come strongly into play.

Finally, the influence of the player on the pitch of a given note is profound. Not only do different players vary widely in the average frequency they produce on a given note, but a single player exhibits a variation in frequencies for different trials.[6] The standard deviation for a single player in several trials is about 6 cents; the standard deviation among flutists is 11 cents, giving a combined value of about 12.5 cents, or one-eighth of a semitone. The meaning of this standard deviation is that one may expect, on any single trial with a randomly picked flutist, to obtain in one-third of the cases a deviation in pitch more than 12.5 cents flat or sharp. The variability due to the player is one of the major difficulties that Boehm faced in the experiments he made—difficulties which were aggravated by the lack of any rapid quantitative means of measuring frequencies.

5. John W. Coltman, "Resonance and Sounding Frequencies of the Flute," *Journal of the Acoustical Society of America* 40, no. 1 (1966): 99–107.

6. John W. Coltman, "Fifty Flutists Play One Flute," *Woodwind World* 15, no. 2 (March, 1976): 31–33, 40.

Boehm's Über den Flötenbau, *1847*

Quite shortly after the introduction of the cylinder flute, Boehm published his extensive treatise of 1847.[7] In it he describes in detail his conical flute of 1832, the cylinder flute of 1847, and the methods he used to determine their important dimensions. We will extract from this the key information that bears directly on the question of tone-hole placement and the essential steps that Boehm took in his approach to flute design, with comments in the light of present-day knowledge. (References to page numbers cite the original publication.)

Boehm does not specifically state what standard pitch he is designing to when he gives the dimensions of his flutes. However, he mentions (p. 39) that the Berlin orchestra tuning fork is at A–441.6 and that of the Vienna orchestra is at A–440.9, and he gives a table (p. 41) of vibration numbers based on A–440. We thus assume that the dimensions he gives in his work are for flutes close to A–440. The tapered head and the new mouth-hole dimensions were apparently used in all the intonation experiments described (p. 49).

Boehm finds experimentally (p. 49) that a cylindrical tube affixed to his headjoint, with a total length of 606 mm from cork to open end, produces *c'*. If we assume that this is an A–440 scale, the frequency would be 261.6 Hz. (In another discussion [p. 38] not involving a flute, Boehm mentions *c'* as 260 Hz.) Apparently the tube described is simply that, namely a smooth tube with no fingerholes or cavities. Had this experiment been done today, we could calculate from our experience with modern flutists,[8] making corrections for the tone-hole cavities, that a smooth-tube flute of 623 mm, cork to open end, would be required to produce middle *c'* of an A–440 scale. This is 17 mm longer than Boehm's value of 606 mm. There is no mistaking that he indeed meant to say cork-to-end distance, since on page 51 he takes a mouth-hole-to-end distance of 589 mm. It is unlikely that this discrepancy arises from our assumption that he was working to A–440, as this would imply an A–452 pitch, well beyond any pitches mentioned in the text. There is a good possibility that the discrepancy results simply from a difference in playing style—that Boehm used a closer lip-to-edge distance and covered more of the mouth hole than is commonly done today.

Boehm assumes now a "theoretically correct" distance of the cork from

7. *Über den Flötenbau* (see n. 1, above).
8. Coltman, "Fifty Flutists."

the mouth-hole center of 23.5 mm without specifying whence this value comes. In locating the apparent end of the flute, he uses double this distance, on the grounds that a stopped pipe is only half as long as an open pipe for the same resonance frequency (p. 44). While the last statement is true at the resonance frequency, a stopped pipe at other frequencies does not behave at all like an open pipe of twice the length. Nevertheless, the value of 47 mm which results is not far from correct. Adding this value to the distance of 589 mm from the mouth hole to the end, Boehm gets 636 mm as the ideal air column length for *c′*, which can be shortened in accordance with the rule for equal-tempered string lengths to get lengths for other notes. In particular, the column length for *c″* is half of this, or 318 mm. To get the lengths of the tubes measured from the cork, Boehm subtracts 23.5 mm, and then, pointing out that his practical cork distance from the mouth hole is actually 17 mm rather than 23.5 mm, he subtracts another 6.5 mm (p. 51). Thus the cord-to-tube-end distances become 318 − 23.5 − 6.5 = 288 mm for *c″*, and 636 − 23.5 − 6.5 = 606 mm for *c′*. The "basic length" is 636 mm, and the total end correction is 30 mm (see fig. 1). Boehm constructed a flute tube with twelve extensions to test the proportions dictated by this calculation, and he seems to have been quite satisfied with the results.

Having now determined how a tube should be cut off to produce the desired scale, Boehm moves to the question of how a side hole behaves, relative to the behavior of a cut-off end. He is quite explicit about a complication that always enters: "for these reasons the tuning of the notes depends not only on the length of the air column alone, but also at the same time on the size of the tone holes, as well as the size and location of the neighboring holes lying below" (p. 53). He proceeds to compare, by experiment, the result of boring a hole in the side to produce C sharp, with the calculated shortening of the tube prescribed in the paragraph above. He does this for holes 12 mm in diameter, with different wall heights, and apparently for several different notes. No mention is made of a key, though the C-sharp hole always has one. His results are very surprising in the light of present knowledge. He finds (p. 54) for a hole of 12 mm diameter with 1 mm minimum chimney height, in a 19 mm tube, that the hole for C sharp must be placed 5.8 mm closer to the mouth hole than the end of a tube which would produce the same C sharp. We know now, with good precision, that this number should be 9.9 mm, rather than 5.8 mm, for a hole of this size. Now Boehm goes on to state (p. 55) that the displacement required increases with each new section, so that for *d′* it is stated to be 6.6 mm, and at the octave *c″* the required displacement is stated to be 12.5 mm. It is completely mysterious what led Boehm to this conclusion. In fact, the displacement required depends only on the effective acoustic length of tube remaining below the open hole. If we assume that for each note, the effective remaining tube is such as to lower the pitch one semitone on closing the hole—which is the case in the flute and presumably in the experiments which Boehm made—then the required displacement *decreases* for higher notes, attaining the value of 7.7 mm at *c″*. Thus, what Boehm found as a stretching of the distance between *c′* and *c″* holes (compared to a simple cutting-off of the tube) of 12.5 − 5.0 (for *c′*) = 7.5 mm should, in fact, be a shrinking of 7.7 − 10.0 = −2.4 mm. The error introduced by this process is close to 10 mm.

Boehm proceeds to apply these (erroneous) corrective displacements of −5.0 and −12.5 mm to the positions for *c′* and *c″* to get 601 and 275.5 mm, respectively, for the distances from the cork to the center of the *c′* and *c″* holes. The 325.5 mm between these must, Boehm reasons, be one-half of a new basic length of 651 mm. Since the *c′* hole is 601 mm from the cork, the reference point must lie 50 mm beyond the cork. He now has, for a flute of unspecified pitch with 12 mm finger holes (no keys), a basic length of 651 mm and a total end correction of 50 mm. The large change in the total end correction results from including both end corrections in this value, the end correction for a side hole being much larger than that for a cut-off end. The frequencies referred to in the publication make it likely that Boehm's working pitch was somewhere around A–440. The corrections that Boehm applied for a side hole, however arrived at, contribute to the "stretch" in the basic length.

Boehm's "Schema" of 1862

Much has been written about Boehm's "schema," a geometrical diagram which provided a way to find the positions of the tone holes for a flute of any desired pitch. While the construction of the schema is cleverly thought out, and it was probably easier to use than making numerical calculations, it was simply a graphical expression of the principles that Boehm had already set forth in his 1847 publication. Its essential parameters are the total end correction, which Boehm considered unvarying with frequency, and a basic length which he specified at one frequency and changed inversely with frequency for different pitches.

The schema was first made public in a submission to the London International Exhibition in 1862,[9] some fifteen years after the publication of *Über den Flötenbau.* In 1859 the French had established their standard pitch of A–435, somewhat lower than what was generally in use in the rest of Europe, and Boehm's submission was intended to make it convenient for instrument makers to redesign their instruments to this pitch, or any other that was desired. Diagrams were presented for the flute, oboe, and clarinet, though the flute was used as an example to explain the construction.

In the explanation which accompanied the submission, Boehm engages in a preliminary illustrative discussion of the principles in which he states: "If now, for example, in order to produce *c′* at the new Paris tuning, a cylindrical flute tube of 20 mm diameter, open on both ends, and 630 mm long is required, then. . . ."[10] This is one of the very few places where Boehm states exactly all the parameters involved. We can calculate with precision, from modern measurements, that a tube of 20 mm diameter, open on both ends and resonating at 258.7 Hz (*c′* for A–435) at 70°F., must be close to 651 mm long. This large discrepancy emphasizes the problems in tracing Boehm's development of his method. This particular discrepancy does not, however, enter into the calculations described immediately below.

As a starting point, Boehm uses the dimensions of "my flute," which he says is built to Munich orchestra pitch, without stating what that pitch was. The dimensions listed are identical with those prescribed in the 1847 publication, using a basic length of 651 mm and a total end correction of 50 mm. He then states that it is necessary to withdraw his headjoint very nearly 8 mm to play *a′* at 435 Hz, the new Paris standard. Adding the 50 mm total end correction to each distance gives a ratio of new-to-old column lengths for *a′* of 395/387.1. In effect he applies this ratio to his old basic length of 651 mm to get a basic length for A–435 of 664.3 mm. He leaves the total end correction at 50 mm. His schema of 1862 is based on these numbers, for A–435.

9. Theobald Boehm, "Schema zur Bestimmung der Löcherstellung auf Blasinstrumenten," submitted to the London International Exhibition of 1862. The submission and the juror's report are in the collection of Karl Ventzke, Düren, Germany. A typed transcription was published by Ludwig Böhm, Munich, 1981.

10. My translation from Ludwig Böhm's transcription of Theobald Boehm's "Schema," p. 10 (see n. 9, above).

We can work this process backward and infer a pitch for "my flute." This turns out to be very nearly A–444, which is presumably the Munich orchestra pitch referred to. However, the evidence in the 1847 publication suggests that Boehm's work some fifteen years previously was carried out at a pitch close to A–441.

The dimensions discussed in Boehm's 1862 explanation offer the only opportunity for a direct comparison of his experience and modern experience. Boehm is explicit in stating that he had to withdraw "my flute" 8 mm in order to sound A–435, and we know that the distance from the cork to the A hole is then 345 mm. From the mouth hole, this is 328 mm. Such a flute, played with today's technique, would sound an *a′* close to 442 Hz, not 435 Hz. We know this not only from direct experiment,[11] but also from the design of modern flutes, which use a distance close to 330 mm for A–440. The evidence here is strong that Boehm's playing technique favored a lip position covering more of the mouth hole than modern flutists use.

Boehm's "Schema" of 1867

Five years after the London Exposition, Boehm submitted his schema to the World's Exposition in Paris. This diagram is slightly modified from that of five years earlier. It uses a total end correction of 51.5 mm and a basic length of 670 mm (A–435) in comparison to the earlier values of 50 mm and 664.3 mm, respectively. The text and explanation of the diagram were published in Munich in 1868.[12]

Here Boehm is more explicit about his basic length. He says, "as a unit of calculation for the longitudinal measurements, I have taken a cylinder open at both ends, 670 mm in length, and giving the note C^1 = 517.3 vibrations of the French normal pitch."[13] (In the notation used in this article we would say *c′* = 258.65 Hz.) Note how different this is from the quotation from the 1862 explanation which called for a 630 mm tube. Perhaps there is a confusion here between a real cylinder open at both ends and a half-wavelength in the tube, the difference being that the real cylinder is effectively longer by the two end corrections of 5.8 mm each. He goes on to discuss the flattening influence of

11. Coltman, "Fifty Flutists."

12. (See n. 2, above.) An English translation by W. S. Broadwood was included in his *An Essay on the Construction of Flutes* (London: Rudall, Carte and Co., 1882), pp. 62–69.

13. Broadwood, *An Essay*, p. 64.

the cork and embouchure (total end correction), assigning to it a value of 51.5 mm. He then states that the length for the octave must be ascertained either by calculation or by shortening the tube, following which it can be doubled to get the air column which corresponds to the string of a monochord. He then describes the diagram and its use in detail, but provides no justification for the choice of these particular basic parameters.

Boehm's Die Flöte und das Flötenspiel, *1871*

In his final treatise on the flute, Boehm reviews extensively much of his earlier work. When he comes to hole placement, he goes directly to the two basic parameters: "It will be found that the actual length of the air column (and therefore also of the flute tube) from the center of a C_3 [*c*′] hole bored in the side of a long flute tube to the face of the cork is 618.5 mm, and that the length of the first octave from the center of the hole for C_3 [*c*′] to the center of the hole for C_4 [*c*″] is 335 mm; thus the upper portion is 51.5 mm shorter than the lower, and this quantity (51.5 mm) must be taken into consideration in calculation. By doubling the length of the octave, one obtains as the theoretical air column the length of 670 mm."[14] The accompanying table of frequencies and lengths makes it certain that he is working now at A–435.

It is not clear whether Boehm actually made this experiment to find the values, or whether the use of the phrase "it will be found" implies that he is only using this description as a straightforward way to make the situation clear, and that he is depending on his experience with actual flutes to supply the numerical values. In making such an experiment, one would have to decide what to put below the hole. Just any length of tube certainly will not do. In particular, to bore the *c*′ hole to get the first value, and then to bore a *c*″ hole about half-way up the tube would be particularly nonsensical; the latter would only act as a vent for the second mode, a fact of which Boehm was certainly aware. The process makes sense for flute design only if the amount of tube below the hole is such as to lower the pitch one semitone on closing the hole.

Using the results from our experience with a large number of flu-

14. Boehm, *Die Flöte und das Flötenspiel,* trans. Miller, *The Flute and Flute-Playing,* p. 34 (see n. 3, above).

tists,[15] together with modern methods of calculation,[16] we can estimate what such an experiment would yield if performed as Boehm describes it, adding the assumption made in the paragraph above. We find that if the experiment were performed with a smooth tube, a distance from cork to hole of 618.3 mm would be required to produce *c′* on an A–435 scale, essentially identical to the value prescribed by Boehm. For *c″*, however, the calculated value is 288.1 mm, some 4.6 mm longer than Boehm's value. The corresponding basic length becomes 660.4 mm, with a total end correction of 42.1 mm. If a real flute were used, in which the cavities under closed keys come into play, a basic length of 650.6 mm and a total end correction of 37.2 mm would result. These calculations assume a note hole of 13.2 mm diameter, provided with a key having a 3 mm rise.

Thus, Boehm's prescription differs rather considerably from what present-day experience would dictate. In the subsequent portion of this article we will address the questions of how Boehm applied his prescription, what actual behavior his flutes exhibit, and how later flutes by other manufacturers depart from the design parameters put forth by Boehm.

Basic Parameters of Specimens of Boehm's Flutes

One of the difficulties in comparing Boehm's flutes with Boehm's prescription lies in the fact that the flutes exhibit quite a variety of tone-hole sizes. It is obvious that if one changes the size of a note hole (or the rise of the key or the height of the rim), it will change the pitch of the note, and if one wishes to compensate for the change, its position must be moved. Boehm says, "The tone holes must therefore be placed nearer the mouth hole, the smaller their diameter and the higher their sides,"[17] but he does not say what rule he followed. In the first two descriptions, he puts forth his schema as a means of determining the proper positions for the centers of the tone holes, without specifying the size of the holes. In his last description, he says he used 13.0 mm holes for wooden flutes and 13.5 mm holes for silver flutes. Actual specimens show frequent deviations from this, and Miller quotes a let-

15. Coltman, "Fifty Flutists."

16. John W. Coltman, "Acoustic Analysis of the Boehm Flute," *Journal of the Acoustical Society of America* 65 (1979): 499–506.

17. Boehm, *Die Flöte und das Flötenspiel,* trans. Miller, *The Flute and Flute-Playing,* p. 26.

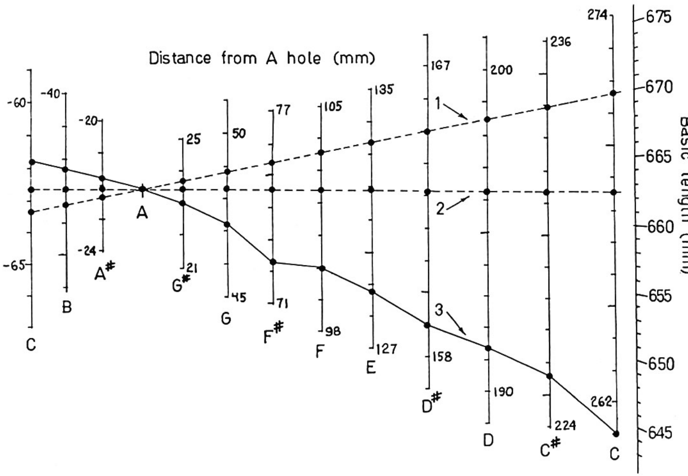

FIGURE 2. Expanded coordinate system. Illustrated are hole positions for:

1. An A–435 flute according to Boehm's 1867 schema
2. An A–440 flute according to Boehm's 1867 schema
3. Modern "new scale" A–440 flute with hole positions adjusted to compensate for varying hole diameter.

ter from Boehm in which Boehm says that for six years (sometime prior to 1862) he made all his silver flutes with graduated tone holes, varying from 12 mm for *c″* to 15 mm for *c′*.[18] Nowhere does he mention how the center of the hole should be moved to compensate.

In comparing Boehm's flutes, I have found it convenient to use a special set of coordinates (fig. 2) on which one can plot directly the distance of each note-hole center from that of the A hole, which is taken as a reference point. The coordinates are chosen in such a manner that the dimensions for any flute constructed in strict accordance with Boehm's schema will give a plot which is a straight line passing through the A coordinate, with a slope that depends on the basic length as defined in figure 1. The total end correction does not appear in this diagram—its apparent value may be found by measuring the distance from the cork to the A hole and combining this suitably with the basic length. The diagram greatly expands the scale of the schema, so that depar-

18. Ibid., p. 28.

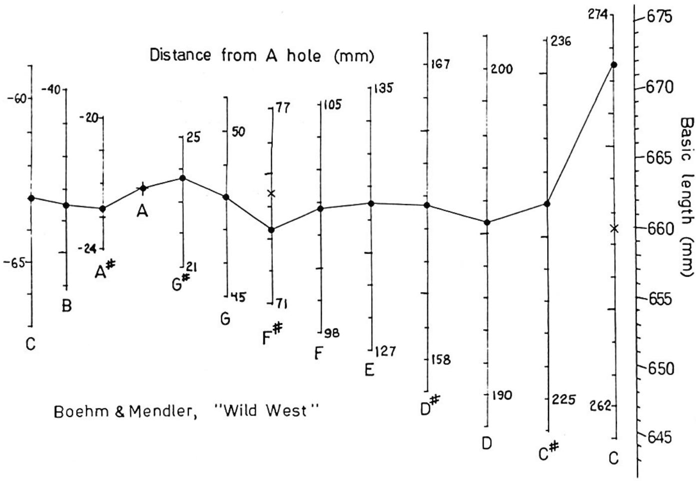

FIGURE 3. Expanded plot for a Boehm and Mendler flute having constant tone-hole size of 13.2 mm.

tures of a fraction of a millimeter are quite evident. A scale at the right-hand side gives the basic length for any straight line passing through the zero point at the A hole. In this figure points are plotted for two of Boehm's designs, as well as for a modern "reference flute," described below.

An example of a real flute is given in figure 3. This is a plot for specimen no. 134 in the Dayton C. Miller Collection, made by Boehm and Mendler (1862–81) and played in Buffalo Bill's Wild West show. All of the major tone holes are 13.2 mm in diameter. On the plot, the points are connected by lines to aid in visualizing the departures.

It will be seen that with the exception of the F-sharp and C holes, the points lie within less than a millimeter of a straight line. The exceptions are easy to explain. Miller notes that the F sharp, when played with the fingering Boehm devised, has one closed hole below the open F-sharp hole, which flattens this note. The F-sharp hole was accordingly moved about 1.2 mm up the tube, which implies in this case a normal position given by the point marked with **X**. Such a correction is appropriate if F sharp were played with the middle finger; it is too large for the usual third-finger fingering, and most flutes today use a smaller correction. The 1.2 mm is typical of specimens of Boehm and Mendler manufacture. The point for *c′* is the location of the end of the flute, not the center of a C hole. As mentioned above, Boehm states that the displacement required (for a 12 mm hole) is 5 mm, and if we make this correction, we get a point close to the dashed line. But, in fact, as discussed above, a 5 mm displacement is incorrect. It should be, for a 13.2 mm hole with key, about 12.5 mm. The interval between *c′* and *c′ sharp* on Boehm's C-foot flutes is consequently always less than a semitone.

One can estimate the basic length to which this flute was designed by drawing a "best line" through the points. A line parallel to this, drawn through the zero at A, will indicate the basic length on the scale at the right. For this flute, we get a value of 661 mm. This corresponds to an A–441 design according to the 1868 schema. The distance from the cork to the A hole, allowing 3 mm withdrawal, is 346.5 mm, from which we derive a total end correction of 46.5 mm, about 5 mm short of the 51.5 mm that Boehm specifies. Playing tests on this instrument (described below) give a tuning *a′* of 440 Hz, though the scale is distorted. It is a reasonable assumption that this is a flute Boehm designed according to his 1868 schema for a pitch close to A–440, though the total end correction is a bit short.

Figure 4 shows plots for two flutes where the results are not so straightforward. The one with the solid line is for an early flute (Th. Boehm no. 21, 1848; Miller no. 653) having graduated tone holes. The estimated basic length for this instrument is 657 mm, with a total end correction of 52 mm.

If we compare this flute with the dimensions prescribed in the publication of 1847, we find that the cork-to-center distance for the upper C hole is essentially unchanged, while the distance to the other holes increases, more or less uniformly, to about 3 mm more at the lower end. One could ascribe this to a flute body designed for a lower pitch, but it is more likely that this stretching results from Boehm's correction for a variable hole size—larger holes being moved farther down the tube. Boehm's publications do not mention flutes designed to different pitches until 1862, where he refers to "my flute" as having only one pitch design.

We see a correction for hole size being applied in the other flute in figure 4, the "Macauley" flute, built in 1877 (Miller no. 160). The tone holes on the body are 13.2 mm in diameter, and those on the foot joint (D sharp to C) are 14.6 mm. The diagram clearly shows that these last four points have been moved down the tube from the line that the

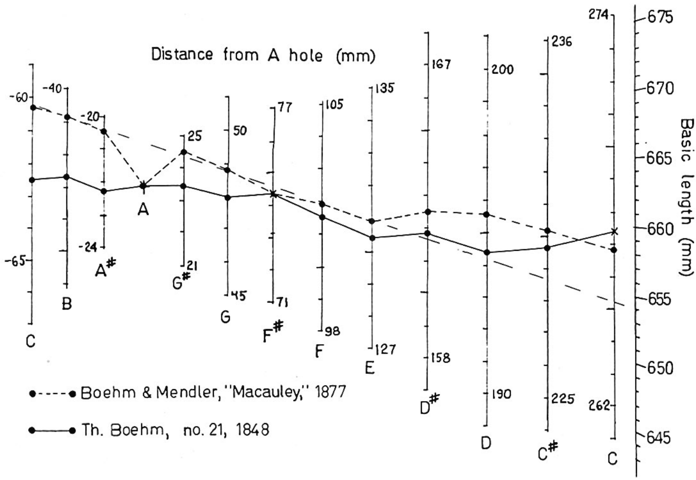

FIGURE 4. Expanded plots for two Boehm flutes.

schema would dictate in order to compensate for their larger diameter. The amount of movement varies from about 1.0 to 1.5 mm. The plot also shows a 1.5 mm displacement of the A hole. The reason for this becomes clear when the G-sharp mechanism is examined. Macauley evidently wanted a closed G sharp; to accomplish this, Boehm did not add a side hole as in the usual flute today, but left the G-sharp key normally closed, with a mechanism to lift it when the G-sharp lever was depressed. Thus, the A would be played with the hole below it closed. To overcome the flattening effect, Boehm evidently moved the A hole 1.5 mm up the tube.

Taking these departures into account, the "best line" (shown dashed in fig. 4) yields a basic length of 651 mm. If we assume that Boehm's 1868 schema was used, this corresponds to an A–448 flute design. The total end correction is 51.2 mm, almost exactly the 51.5 mm that Boehm specifies. Playing tests on this flute agree quite well with the A–448 for notes near the tuning note, though the *a′* itself is somewhat flat. Calculations using modern techniques indicate that in order to compensate for the closed G sharp, the A hole should have been moved 3.7 mm, rather than 1.5 mm. Similarly, we find that for the change in hole size on the foot keys, a movement of 3 mm would be needed rather than the 1.0 to 1.5 mm that Boehm used.

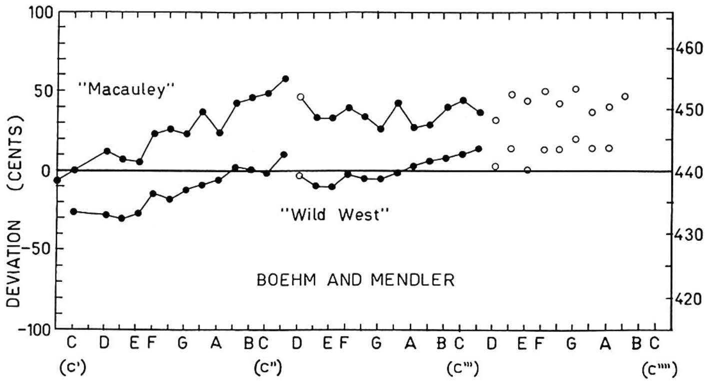

FIGURE 5. Observed deviations from the equal-tempered scale of two Boehm and Mendler flutes. (A semitone is 100 cents.)

Plots like these have been made for several specimens of Boehm's instruments. We conclude from them that Boehm used his schema, at least in later years, as a starting point for the design, but moved the holes (by modest amounts) to compensate for diameter changes. He does not appear to have made a distinction between holes with keys and holes without keys, as found in his earliest models, though properly such a correction should be 2.5 to 3 mm in position. The irregularities in the plots and the lack of definite knowledge of the design pitch for a particular instrument prevented derivation of the rules that Boehm used for compensating for hole size, something that he clearly did. Aside from mentioning its necessity, he is silent on the subject.

Intonation Characteristics

So far, we have tried to establish what it was that Boehm intended to do and the methods he employed to carry out these intentions. We turn now to what he actually produced, and compare it with later developments in the instruments.

The actual intonation characteristics of two of Boehm's flutes, the "Wild West" and the "Macauley," are shown in figure 5.[19] The graph

19. John W. Coltman, "The Intonation of Antique and Modern Flutes," *The Instrumentalist* 29, no. 5 (December, 1974): 53–55; vol. 29, no. 6 (January, 1975): 43–47; vol. 29, no. 7 (February, 1975): 47–52; and vol. 29, no. 8 (March, 1975): 77–80.

clearly displays the distorted scale of the first register, flat in the lower notes and sharp in the upper ones, that results from using a basic length and a total end correction that are too long. To make comparisons of instruments by playing them requires an enormous number of readings with many players and is still subject to many uncertainties. The differences can be more certainly established by mechanical measurement of the essential parameters. We therefore make a comparison of the center positions of the tone holes of various instruments after their positions have been corrected, using modern methods, to a standard hole size. This hole size was chosen to be 13.2 mm; more accurately an "effective height"[20] of 27.5 mm was used as a standard, and the actual hole correction took into account the height of the rim, size of the key cover, rise, etc. Our reference flute, to which all others will be compared, is an average of several "new scale" A–440 instruments of recent American, English, and Japanese manufacture, which do not in fact differ greatly. The relative hole positions for this reference flute are plotted on the "schema" diagram as curve 3 in figure 2, where it will be seen that they follow a gently curved, rather than straight, line. Using these center positions, measured from the center of the mouth hole, and a suitable logarithmic function for other frequencies, the coordinate system of figure 6 has been worked out. Plotted on these coordinates, our reference flute gives a straight horizontal line through A–440. A flute scaled for another frequency will give a horizontal line through the new design frequency, as read from the scale on the A line. A point which departs from the line indicates a finger hole which is out of position (or too large or too small) compared to its "proper" placement as defined by the reference flute. We must not take the results too seriously as regards the average position of any curve, since withdrawal of the head joint, differences in size and height of the embouchure hole, etc., can easily make a change of a few millimeters. But the irregularities, and the general slopes, are significant.

The solid curve in figure 6 is for a representative Boehm and Mendler flute, the "Wild West" flute described above. The curve has a marked slope; compared to the reference flute it is "stretched" almost 8 mm over the length from *c′* to *c″*. The dashed curve is for a Rudall, Carte and Co. flute built in 1898 (Miller no. 5). It too shows a stretch compared to today's "new scale" instruments, but one that averages about 3 mm.

20. The definition of this term is given in Coltman, "Acoustic Analysis."

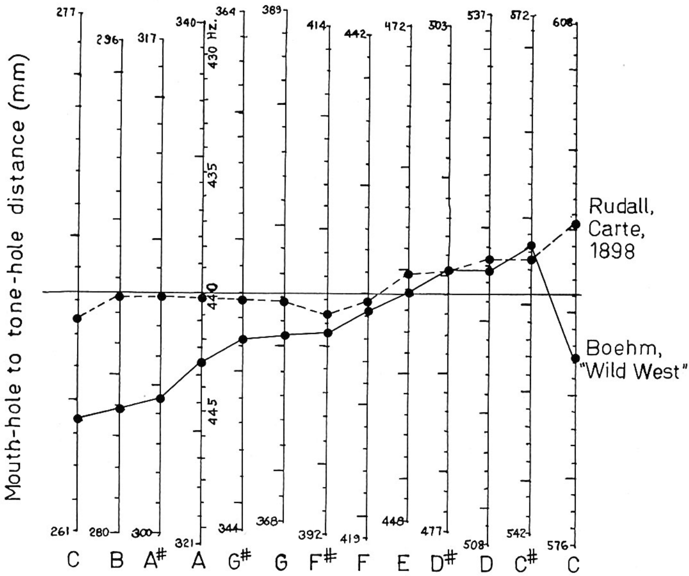

FIGURE 6. Coordinate system for displaying tone-hole positions after adjustment to a standard hold size. The horizontal line at A–440 represents the "new scale" reference flute; the plotted points are for two actual instruments.

In figure 7 are diagrammed a Wm. S. Haynes flute of 1922, and the design of a large American manufacturer, W. T. Armstrong, prior to the changeover to the "new scale." These show slightly more slope than the Rudall, Carte and Co. instrument.

Summary of Flute Measurements

Tables 1 and 2 summarize the results of measuring several of Boehm's instruments and Boehm flutes of other manufacturers made over the last century or so.

In table 1, the first two columns ("Boehm's intended pitch") attempt to specify the pitch to which Boehm designed the instrument, by comparing their measurements with the instructions that Boehm gave—the first five items of the early years relying on his *Über den Flötenbau* (1847)

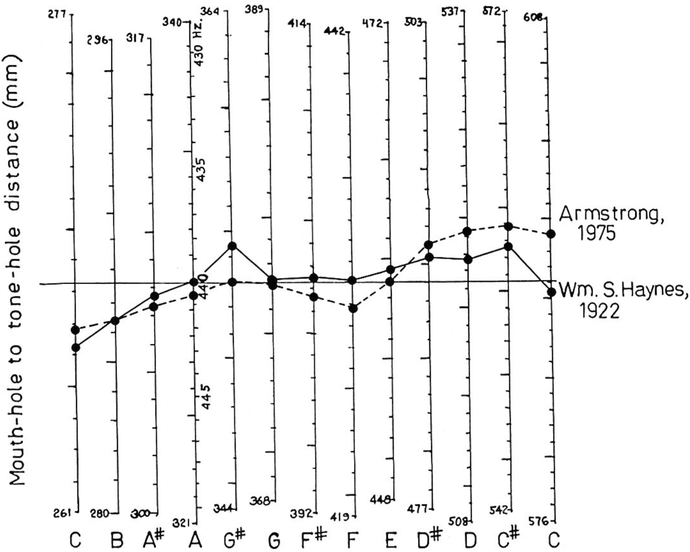

FIGURE 7. Adjusted hole positions of two American flutes, compared to the "new scale" reference flute.

and the second five his schema of 1867. Two separate parameters are used: the distance from the cork to the A hole, and the slopes of lines from plots like figures 3 and 4. It will be seen that quite consistent results are obtained (Boehm no. 4 apparently has an altered headjoint; it is far too short to make sense) between the two parameters. The second two columns give the results that one would expect modern players to obtain when playing the instruments, as taken from plots like figures 7 and 8. The slope of the best line in such a plot has been used to specify a pitch characteristic of the body dimensions, that is, the pitch of the scale produced if the headjoint were adjusted to give a scale most closely proportional to that of the "new scale" flutes in use today.

Looking at the first item, we see that Boehm's 1862 schema (based on "my flute" of 1847) specifies for A–440 an instrument that would have a body suited to A–433, but an A-hole distance that would produce 448 Hz for the tuning note. These departures appear in all the flutes of this early period. Actually, these do not cover a wide range of

TABLE 1

Pitch Characteristics of Theobald Boehm Flutes

	Boehm's intended pitch: from A-hole distance (Hz)	*from basic length (Hz)*	*Estimated sounding pitch of* **a′** *(Hz)*	*Body suited to A (Hz)*
1. Boehm's 1862 Schema	440	440	448	433
2. Boehm, "my flute," 1847–62	444	444	451	437
3. Boehm, no. 4, 1848 (Swain)	?	444	462?	439
4. Boehm, no. 21, 1848 (Miller no. 652)	443	441	450	437
5. Boehm, no. 38, 1849	439	441	449	436
6. Boehm's 1867 Schema	440	440	444	430
7. Boehm and Mendler, 1862–81 ("Wild West"; Miller no. 134)	438	441	441	429
8. Boehm and Mendler, 1862–81 ("Oliver"; Miller Data 57)	437	440	442	430
9. Boehm and Mendler, 1877 ("Macauley"; Miller no. 161)	450	448	452	440
10. Boehm and Mendler, 1877 (Miller no. 233)	435	435	438	424

pitches, and our uncertainty as to how Boehm intended to correct for hole-size variation, and what he allowed for headjoint withdrawal, makes it unclear whether he was really designing flutes to different pitches in this period. In any event, the too-long body and too-short headjoint are strongly evident.

The Boehm and Mendler flutes are characterized under the assumption that they were designed according to the 1867 schema, the results of whose specifications for an A–440 flute are given as item 6. Comparing this with item 1, we see that the new schema ameliorates the too-short head (*a′* sounds 444 Hz rather than 448 Hz) at the expense of an even more stretched body. This shows up consistently in the results for actual specimens. The last one of these can be spotted in design pitch with confidence, since it corresponds to A–435, the lowest pitch in use anywhere, and one for which we know that Boehm offered to make instruments.

Table 2 gives similar figures for flutes of later periods by other manufacturers. We specify only the probable design pitch. The actual meth-

TABLE 2

Pitch Characteristics of Selected Boehm Flutes

	Probable design pitch (Hz)	*Estimated sounded a′ (Hz)*	*Body suited to A (Hz)*
1. Rudall, Carte and Co., 1892 (wood, 1867 model; serial no. 2280)	452	452	450
2. Rudall, Carte and Co., 1896 (ebonite; serial no. 2777; Miller no. 4)	435	435	435
3. Rudall, Carte and Co., 1899 (silver; serial no. 3015; Miller no. 5)	440	440	437
4. C. G. Conn, 1896 (Howe model, ebonite and brass)	440	440	435
5. D. C. Miller, 1905 (gold, 1 piece; Miller no. 10)	435	435	433
6. Wm. S. Haynes, 1922 (silver; serial no. 5611)	440	440	433
7. Cundy-Bettony, 1927 ("Boston Wonder"; serial no. A1318)	440	440	431
8. Armstrong 304, 1975 (student model)	440	440	436
9. "New Scale" Flutes, 1975–82 (standard of comparison)	440	440	440

ods used are unknown, though some of the Rudall, Carte and Co. flutes are consistent with the design method put forth by Rockstro.[21] The design pitches of items 1, 2, and 5 are well established. The use of A–452 (item 1) would be standard for an English military band at this period, and we know from Miller's records that item 2 was a flute he ordered built at A–435, and that the gold flute he made himself (item 5) was his design for an A–435 instrument. These three instruments are very self-consistent. Their performance today (at their design pitches) would compare well with that of the "new scale" flutes which is our standard of comparison.

The A–440 flutes in table 2 all have "stretched" bodies, corresponding to pitches from 431 to 437 Hz. This is about half as large a discrepancy as exhibited by Boehm and Mendler flutes. There is not enough data here to trace the probable origins of these designs.

We have seen in the earlier descriptions of Boehm's work the diffi-

21. Walter E. Worman, "Boehm's Design of the Flute: A Comparison with That of Rockstro," *Galpin Society Journal* 28 (1975): 107–20.

culties that he faced in trying to pin down the many variables, and that certain of his experiments clearly gave erroneous results. These may have influenced Boehm's choice of design parameters. Nevertheless, it seems to me that there must have been a real difference in the way that Boehm himself played the flute, as compared with the blowing technique used by the average American player. We are again reminded that there are no perfectly tuned flutes; there are only flutes suited to their players.

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